Units and Measurement

Fill Ups

Q.1. Planck's constant has dimension ________. (1985 - 2 Marks)

Ans. $ML^2 T^{-1}$

Solution.

$$E = h\nu \qquad h = \frac{E}{\nu} = \frac{[ML^2T^{-2}]}{[T^{-1}]} = [ML^2T^{-1}]$$

Q.2. In the formula X = 3YZ², X and Z have dimensions of capacitance and magnetic induction respectively. The dimensions of Y in MKSQ sytem are _________, ________. (1988 - 2 Marks)

Ans. $M^{-3} L^{-2} T^4 Q^4$

Solution.

$$[X] = [C] = [M^{-1} L^{-2} T^2 Q^2]$$
$$[Z] = [B] = [MT^{-1} Q^{-1}]$$
$$\therefore \quad [Y] = \frac{[M^{-1}L^{-2}T^2Q^2]}{[MT^{-1}Q^{-1}]^2} = [M^{-3}L^{-2}T^4Q^4]$$

Q.3. Th e equation of state for real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$ **The dimensions of the constant a is _________.**

Ans. $ML^5 T^{-2}$

Solution.

$$[a] = [PV^2] = \frac{MLT^{-2}}{L^2} L^6 = ML^5T^{-2}$$

Subjective Questions

Q.1. Give the MKS units for each of the following quantities. (1980)

(i) Young's modulus

(ii) Magnetic Induction

(iii) Power of a lens

Ans. (i) N/m^2;

(ii) Tesla;

(iii) Dioptre;

Solution. The M.K.S. unit of Young's modulus is Nm^{-2}.

The M.K.S. unit of magnetic induction is Tesla.

The M.K.S. unit of power of lens is Dioptre.

Q.2. A gas bubble, from an explosion under water, oscillates with a period T proportional to $p^a d^b E^c$. Where 'P' is the static pressure, 'd' is the density of water and 'E' is the total energy of the explosion. Find the values of a, b and c.

Ans. $a = -5/6$, $b = 1/2$, $c = 1/3$

Solution. Given that $T \propto P^a d^b E^c$

$\Rightarrow [M^0L^0T^1] = [ML^{-1}T^{-2}]^a\, [ML^{-3}]^b\, [ML^2T^{-2}]^c$

$\therefore [M^0L^0T^1] = [M^{a+b+c}\, L^{-a-3b+2c}\, T^{-2a-2c}]$

$\therefore a + b + c = 0,\ -a - 3b + 2c = 0$

$-2a - 2c = 1$

On solving, we get

$a = -5/6$, $b = 1/2$, $c = 1/3$

Q.3. Write the dimensions of the following in terms of mass, time, length and charge (1982 - 2 Marks)

(i) magnetic flux

(ii) rigidity modulus

Ans. (i) $[M^1L^2T^{-1}Q^{-1}]$ (ii) $[ML^{-1}T^{-2}]$

Solution. Magnetic Flux = $[M^1L^2T^{-1}Q^{-1}]$

Modulus of Rigidity = $[ML^{-1}T^{-2}]$

Q.4. Match the ph ysical quan tities given in column I with dimensions expressed in terms of mass (M), length (L), time (T), and charge (Q) given in column II and write the correct answer against the matched quantity in a tabular form in your answer book.

Column I	Column II
Angular momentum	ML^2T^{-2}
Latent heat	ML^2Q^{-2}
Torque	ML^2T^{-1}
Capacitance	$ML^3T^{-1}Q^{-2}$
In ductan ce	$M^{-1} L^{-2} T^2 Q^2$
Resistivity	L^2T^{-2}

Solution.. Angular Momentum $[ML^2T^{-1}]$;

Latent heat $[L^2T^{-2}]$;

Torque $[ML^2T^{-2}]$

Capacitance $[M^{-1}L^{-2}T^2Q^2]$;

Inductance $[ML^2Q^{-2}]$;

Resistivity $[ML^3T^{-1}Q^{-2}]$

Q.5. Column -I gives three physical quant ities. Select the appropriate units for the choices given in Column-II. Some of the physical quantities may have more than one choice correct :

Column I	**Column II**
Capacitance	**(i) ohm-second**
In ductan ce	**(ii) coulomb^2–joule^{-1}**
Magnetic Induction	**(iii) coulomb (volt)$^{-1}$**
	(iv) newton (amp-metre)$^{-1}$
	(v) volt-second (ampere)$^{-1}$

Ans. Capacitance coulomb-volt^{-1}, coulomb^2-joule^{-1}

In ductance ohm-sec, volt-second (ampere)$^{-1}$

Magnetic Induction newton (ampere-metre)$^{-1}$

$\because$ (a) $q = CV; U = \frac{1}{2}CV^2$

(b) Refer to solution of Q. 3, type D

(c) $F = I\ell B$

Q.6. If nth division of main scale coincides with (n+1)th divisions of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier. (2003 - 2 Marks)

Ans. a/n + 1 units

Solution. (n + 1) divisions of vernier scale = n divisions of main scale.

$\therefore$ One vernier division = a/n + 1 main scale division

$$\text{Least count} = 1\text{ M.S.D} - 1\text{VS.D} = \left(1 - \frac{n}{n+1}\right)\text{MSD}$$

$$= \frac{1}{n+1}\text{M.S.D.} = \frac{a}{n+1}\text{ units} \quad [\because 1\text{ MSD} = a\text{ units}]$$

Q.7. A screw gauge having 100 equal divisions and a pitch of length 1 mm is used to measure the diameter of a wire of length 5.6 cm. The main scale reading is 1 mm and 47thcircular division coincides with the main scale. Find the curved surface area of wire in cm^{2}to appropriate significant figure.

(use π = 22/7)

Ans. 2.6 cm^2

Solution.

$$\text{Least Count} = \frac{1\text{mm}}{100} = 0.01\text{mm}$$

Diameter = MSR + CSR × (least count)

= 1 mm + 47 × (0.01) mm = 1.47 mm Surface Area = πDl

$$= \frac{22}{7} \times 1.47 \times 56\text{mm}^2 = 2.58724\text{cm}^2$$

= 2.6 cm^2 (Rounding off to two significant figures)

Q.8. In Searle's exper iment, which is used to find Young's Modulus of elasticity, the diameter of experimental wire is D = 0.05 cm (measured by a scale of least count 0.001 cm) and length is L = 110 cm (measured by a scale of least count 0.1 cm). A weight of 50 N causes an extension of X = 0.125 cm (measured by a micrometer of least count 0.001cm). Find maximum possible error in the values of Young's modulus.

Screw gauge and meter scale are free from error. (2004 - 2 Marks)

Ans. 1.09 × 10^{10} Nm $^{-2}$

Solution. $Y = \frac{W}{\frac{\pi D^2}{4}} \times \frac{L}{X}$

KEY CONCEPT : Maximum error in Y is given by

$$\left(\frac{\Delta Y}{Y}\right)_{max} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta X}{X} + \frac{\Delta L}{L}$$

$$= 2\left(\frac{0.001}{0.05}\right) + \left(\frac{0.001}{0.125}\right) + \left(\frac{0.1}{110}\right) = 0.0489$$

It is given that W = 50 N; D = 0.05 cm = 0.05×10^{-2}m;

X = 0.125 cm = 0.125×10^{-2}m;

L = 110 cm = 110×10^{-2}m

$$\therefore\ Y = \frac{50 \times 4 \times 110 \times 10^{-2}}{3.14(0.05 \times 10^{-2}) \times (0.125 \times 10^{-2})} = 2.24 \times 10^{11} N/m^2$$

∴ Maximum possible error in the value of

Y = DY = $0.0489 \times 2.24 \times 10^{11}$

= 1.09×10^{10} N/m^2

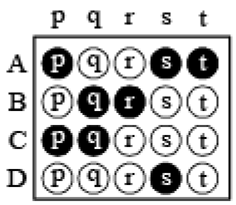
Q.9. The side of a cube is measured by vernier callipers (10 divisions of a vernier scale coincide with 9 divisions of main scale, where 1 division of main scale is 1 mm). The main scale reads 10 mm and first division of vernier scale coincides with the main scale. Mass of the cube is 2.736 g. Find the density of the cube in appropriate significant figures.

Ans. 2.66 gm/cm^3

Match the Following

DIRECTIONS (Q. No. 1) : Each question contains statements given in two columns, which have to be matched. The statements in Column-I are labelled A, B, C and D, while the statements in Column-II are labelled p, q, r and s. Any given statement in Column-I can have correct matching with ONE OR MORE statement(s) in Column-II. The appropriate bubbles corresponding to the answers to these questions have to be darkened as illustrated in the following example :

If the correct matches are A-p, s and t; B-q and r; C-p and q; and D-s then the correct darkening of bubbles will look like the given.



Q.1. Some physical quantities are given in Column I and some possible SI units in which these quantities may be expressed are given in Column II. Match the physical quantities in Column I with the units in Column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. (2007)

Column I	**Column II**
(A) GM_eM_s , G – universal gravitational constant, M_e – mass of the earth, M_s – mass of the Sun	(p) (volt) (coulomb)(metre)
(B) 3RT/M , R – universal gas constant, T – absolute temperature, M – molar mass	(q) (kilogram) $(\text{metre})^3(\text{second})^{-2}$
(C) $\frac{F^2}{q^2B^2}$, F – Force, q – charge, B – magnetic field	(r) $(\text{metre})^2$ $(\text{second})^{-2}$
(D) $\frac{GM_e}{R_e}$, M_e – mass of the earth, R_e – radius of the earth	(s) (farad) $(\text{volt})^2$ $(\text{kg})^{-1}$

Ans. (A) → p, q ; (B) → r, s ; (C) → r, s ; (D) → r,s

Solution. A : p → q

Reason : Unit of $GM_eM_s = Fr^2 = Nm^2$

$= kg\frac{m}{s^2} \times m^2$

$= kg\ m^3s^{-2}$

Also (volt) (coulomb) (metre) = (joule) (metre)

= (N - m) (m) = $Nm^2 = kg\ m^3s^{-2}$

B : r → s

Reason : $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms}^2 = \frac{3RT}{M}$

$\Rightarrow$ Unit of $\frac{3RT}{M}$ is $m^2 s^{-2}$

C : r → s

Reason : $F = qvB \Rightarrow v^2 = \frac{F^2}{q^2B^2}$

∴ Unit of v^2 is m^2s^{-2} which is further equal to $FV^2\ kg^{-1}$.

D : r → s

Reason : Escape velocity $v_e = \sqrt{\frac{2GM}{R}} \Rightarrow v_e^2 = \frac{2GM}{R}$

∴ The unit of $\frac{GM}{R}$ is $m^2 s^{-2}$.

DIRECTIONS (Q. No. 2) : Following question has matching lists. The codes for the lists have choices (a), (b), (c) and (d) out of which ONLY ONE is correct.

Q.2. Match List I with List II and select the correct answer using the codes given below the lists: (JEE Adv. 2013)

List I	**List II**
P. Boltzmann constant	1. $[ML^2T^{-1}]$
Q. Coefficient of viscosity	2. $[ML^{-1}T^{-1}]$
R. Planck constant	3. $[MLT^{-3}K^{-1}]$
S. Thermal conductivity	4. $[ML^2T^{-2}K^{-1}]$

Codes:

	P	**Q**	**R**	**S**
(a)	3	1	2	4
(b)	3	2	1	4
(c)	4	2	1	3
(d)	4	1	2	3

Ans. (c)

Solution.

$$\text{Boltzmannn constant} = \frac{R}{N} = \frac{PV}{nTN} = \frac{ML^{-1}T^{-2} \times L^3}{K}$$
$$= ML^2T^{-2}K^{-1}$$

$$\text{Coefficient of viscosity} = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{L \times LT^{-1}} = ML^{-1}T^{-1}$$

$$\text{Planck constant} = \frac{E}{\nu} = \frac{ML^2T^{-2}}{T^{-1}} = ML^{-2}T^{-1}$$

$$\text{Thermal conductivity} = \frac{H\ell}{tA\Delta T} = \frac{ML^2T^{-2} \times L}{T \times L^2 \times K}$$
$$= MLT^{-3}K^{-1}$$

(c) is the correct option.

Integer Value Correct Type

Q.1. To find the distance d over which a signal can be seen clearly in foggy conditions, a railways-engineer uses dimensions and assumes that the distance depends on the mass density ρ of the fog, intensity (power/area) S of the light from the signal and its frequency f. The engineer finds that d is proportional to $S^{1/n}$. The value of n is (JEE Adv. 2014)

Ans. (3)

Solution.

$$d \propto \rho^x S^y f^z$$
$$M^0 L^1 T^0 = M^x L^{-3x} M^y T^{-y} T^{-z}$$
$$M^0 L^1 T^0 = M^{x+y} L^{-3x} T^{-y-z}$$
$$\therefore x+y=0,\ -3x=1$$
$$\therefore\ x=-\frac{1}{3} \text{ and } y=\frac{1}{3}$$
$$\therefore n=3$$

Q.2. During Searle's experiment, zero of the Vernier scale lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale. The 20th division of the Vernier scale exactly coincides with one of the main scale divisions. When an additional load of 2 kg is applied to the wire, the zero of the Vernier scale still lies between 3.20×10^{-2} m and 3.25×10^{-2} m of the main scale but now the 45th division of Vernier scale coincides with one of the main scale divisions. The length of the thin metallic wire is 2 m and its cross-sectional area is $8 \times 10^{-7} m^2$. The least count of the Vernier scale is 1.0×10^{-5} m. The maximum percentage error in the Young's modulus of the wire is (JEE Adv. 2014)

Ans. (4)

Solution.

$$Y=\frac{FL}{a \times l}$$

Here F, a and L are accurately known.

$$\frac{\Delta Y}{Y}\times 100=\frac{\Delta L}{l}\times 100=\frac{1.0\times 10^{-5}}{25\times 10^{-5}}\times 100=4\%$$

Q.3. The energy of a system as a function of time t is given as E(t) = A^2 exp(–αt,) where a = 0.2 s^{-1}. The measurement of A has an error of 1.25%. If the error in the measurement of time is 1.50%, the percentage error in the value of E(t) at t = 5 s is (JEE Adv. 2015)

Ans. (4)

Solution. E = A^2 e–0.2t

∴ $\log_e$ E = 2 loge A –0.2t

On differentiating we get

$$\frac{dE}{E} = 2\frac{dA}{A} - 0.2\frac{dt}{t} \times t$$

As errors always add up therefore

$$\frac{dE}{E} \times 100 = 2\left(\frac{dA}{A} \times 100\right) + 0.2t\left(\frac{dt}{t} \times 100\right)$$

$$\therefore \quad \frac{dE}{E} \times 100 = 2 \times 1.25\% + 0.2 \times 5 \times 1.5\%$$

$$\therefore \quad \frac{dE}{E} \times 100 = 4\%$$